

LAMINAR FLOW PAST A CIRCULAR CYLINDER UNDER THE EFFECT OF NONSTATIONARY JET EFFLUX TO THE NEAR-WAKE REGION

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Changes in a laminar flow past a circular cylinder under the effect of jets blowing out to its rare region have been studied numerically. Jet efflux was caused by a return flow of a part of the incoming flow in the inner channels of the cylinder due to the pressure difference between its front and end stagnation points. The channels connected the inlet window formed in the zone of the front stagnation point with two outlet windows on the backside surface of the cylinder. The influence of the coordinates of the jets issuing through the outlet windows and dimensions of the inlet and outlet windows on variation of the drag and lift coefficient and distribution of the coefficients of pressure and friction on the cylinder surface has been considered. It is shown that under the effect of blowing out the cylinder drag decreased by 4% and the amplitude of buoyancy force oscillations decreased by 40%.

Introduction. The efficiency of monitoring the drag of poorly streamlined bodies and attenuation of wake oscillations by a jet issuing to the near-wake region is shown in [1–4]. Methods of reducing the amplitude of a normal alternating force generated by nonstationary separation of boundary layers are of interest for huge engineering constructions, e.g., towers, piers, and sea pipelines, since these methods directly affect the period of their reliable operation. It is evident that for such objects it is most rational to use methods that do not cause substantial complication of objects' structures and do not require additional energy consumption. Blowing out of jets under the effect of the pressure difference between the front stagnation point of the flow and the rarefaction zone arising in the rear region of poorly streamlined bodies is one of the few methods that satisfy the requirements mentioned. The experimental [1, 2] and numerical studies [5] available in the literature on the effect of nonstationary blowing out on aerodynamic characteristics of the object are limited either to investigation of the effect of only one central jet or the fixed angular coordinate of two symmetric jets. An analysis of variation of the drag and lift coefficients as a function of the position of blown-out jets and their flow rates is absent. To gain a full picture of flow transformation around the cylinder, of interest are the distributions of the coefficients of pressure and surface friction around the contour of the cylinder. Measurement of all parameters at the coordinated instants of time is a very complex problem for an experiment but is quite attainable in numerical calculation. As is known, for the Reynolds numbers $Re > 190$ the flow behind the cylinder becomes three-dimensional and its calculation becomes more complicated. However, nonstationarity arises behind the cylinder even when $Re > 40$, i.e., in a laminar flow past the cylinder, and at present this regime can be calculated with high accuracy. This paper is devoted to numerical investigation of the influence of nonstationary efflux of jets on drag, buoyancy force, and distribution of pressure and surface friction around the contour of a circular cylinder depending on the angular coordinates and flow rates of jets.

Numerical Method and the Studied Model. We consider a laminar Newtonian viscous incompressible fluid flow in the absence of the effect of mass forces and at $Re = 150$. In this case, a nonstationary flow is adequately described by the Navier–Stokes equations within the framework of the two-dimensional approach:

$$\frac{\partial \mathbf{V}}{\partial t} = -\nabla(\mathbf{V}\mathbf{V}) + \nabla[\nu(\nabla\mathbf{V} + \nabla\mathbf{V}^t)] - \nabla p. \quad (1)$$

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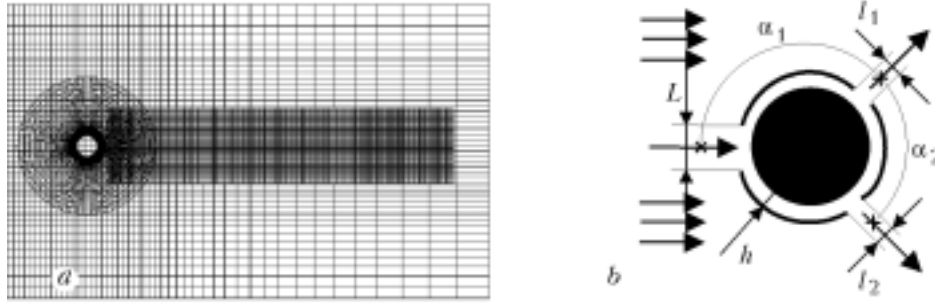


Fig. 1. Composite grid (a) consisting of rectangular grids of the computational domain, subregion in the wake, and two cylindrical grids around the cylinder and the cylinder with internal channels (b).

The equation of momentum transfer is supplemented by the continuity equation

$$\nabla \mathbf{V} = 0. \quad (2)$$

Equations (1) and (2) are written in dimensionless form. The coordinates are nondimensionalized over the characteristic size L , the velocity components — over some characteristic velocity U , pressure — over the double velocity head ρU^2 , viscosity — over the complex UL , and time — over the characteristic time that is determined as a ratio of the characteristic scale to the characteristic velocity. Special features of the solution are given in [5, 6].

The outer boundaries of the computational domain are at a rather large distance from the body contour. On the inlet boundary, fixed parameters of the oncoming flow, which are the normalization parameters, are specified and on outlet boundaries, soft boundary conditions are specified. The adhesion conditions are realized on the washed surfaces of the cylinder.

A transverse flow past the cylinder was calculated on a composite grid [6]. The outer rectangular grid with a size $(27 \times 20)D$ had 70×60 meshes (Fig. 1a). The minimum mesh width near the cylinder contour is $0.2D$. The cylinder was surrounded by a two-layered cylindrical grid. The annular zone adjacent to the cylinder surface had a thickness of $0.2D$ and 15×100 meshes with the minimum mesh width near the surface being $0.005D$. The outer layer extended to a distance $2D$ and had 20×100 meshes. For a detailed description of the wake behind the cylinder we used an additional rectangular grid of length $13D$ and width $3D$. The grid had 175×40 meshes and its minimum mesh width was $0.05D$. The flow in the internal channels was calculated by an annular grid of width $0.1D$ and with number of meshes 20×150 . The near-wall and minimum mesh widths in the vicinity of a sharp edge were equal to $0.005D$.

The object of the study was a circular cylinder with internal channels that connected the front stagnation point of the oncoming flow (the zone of elevated pressure) with the windows on the leeward side of the cylinder (the rarefaction zone) (Fig. 1b). The channels are made along the arc concentrically to the outer contour of the cylinder, they started in the window coinciding with the front stagnation point, had the same configuration and height ($h_1 = h_2 = 0.1D$), and ended by the outlet windows that were symmetric relative to the front critical point ($\alpha_1 = -\alpha_2$). The size of the inlet window around the cylinder contour L is double the size of the outlet windows $l_1 = l_2$ through which the medium escapes. Calculations are made for the cylinder with the inlet window of dimensions $L = 0.1D, 0.2D,$ and $0.3D$ at constant angular coordinates of the outlet windows: 1) $\alpha_1 = 110, \alpha_2 = 250^\circ$; 2) 130 and 230° ; 3) 140 and 220° ; 4) 160 and 200° ; 5) 170 and 190° , and 6) $\alpha_1 = \alpha_2 = 180^\circ$. In the latter case, the inlet and outlet windows had the same size.

Influence of Blowing-out on Aerodynamic Characteristics of the Cylinder. *Variation of the drag and lift coefficients.* The computational complex was verified for the basic cylinder — a circular cylinder of diameter D without internal channels. The calculated mean value of the coefficient $C_x = 1.341$ is in agreement with experimental data within 2% [7] and 12% [8, p. 31].

Asymmetric separation of the boundary layer from the opposite sides of the cylinder causes alternating variation of the lift coefficient C_y from -0.52 to 0.52 (root-mean-square deviation $C'_y = 0.352$) and periodic variations of

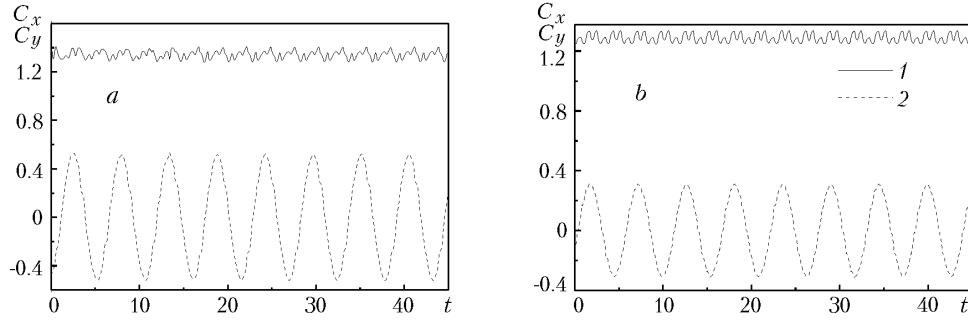


Fig. 2. Variation of the drag and lift coefficients of the cylinder with time: a) blowing-out is absent [1] $C_x = 1.316$, $C'_x = 0.0316$; 2) $C_y = -0.532-0.522$, $C'_y = 0.352$; b) jet action, $L = 0.3D$, $\alpha_1 = 160^\circ$, $\alpha_2 = 200^\circ$ [1] $C_x = 1.288$, $C'_x = 0.028$; 2) $C_y = -0.311-0.312$, $C'_y = 0.216$.

the drag coefficient relative to the mean value $C_x = 1.341$ with a frequency much higher than the frequency of variation of the buoyancy force (Fig. 2a). The period of variation of the buoyancy force amplitude, which is determined on a self-oscillation section, i.e., where the values of the amplitude are constant, was used to determine the Strouhal number $St = 0.185$. This value virtually coincides with the value $St = 0.184$ calculated by the relation that approximates experimental data for the range of Reynolds numbers $50 \leq Re \leq 160$ [9]

$$St = -3.3265/Re + 0.1816 + 1.6 \cdot 10^{-4} Re.$$

Root-mean-square values of oscillations of the lift coefficient (C'_y) within the range $Re_{cr} < Re < 260$ can be estimated from the relation [10]

$$C'_y = (\varepsilon/30 + \varepsilon^2/90)^{0.5},$$

where $\varepsilon = (Re - Re_{cr})/Re_{cr}$, $Re_{cr} = 47$.

In the considered range of Reynolds numbers, $C'_y = 0.354$, which is in agreement with the value obtained in our calculation.

Efflux of the medium through the outlet windows of the cylinder to the near wake led to variation of both local and averaged values of the coefficients C_x and C_y (Fig. 2). The cylinder drag and maximum value of the coefficient C_y decreased. An increase in the flow rate of the issuing jets (an increase in the parameter L from $0.1D$ to $0.3D$) amplified the observed tendency [11].

The values of the drag coefficient and buoyancy force of the cylinder were averaged on the self-oscillation section. Within the considered range of Reynolds numbers, a laminar boundary layer with separation coordinates $\sim 115^\circ$ is formed on the basic cylinder [12, p. 89]. Blowing-out of jets at the angular coordinates of the outlet windows $\alpha_1 = 110^\circ$ and $\alpha_2 = 250^\circ$ is likely to cause earlier separation of the boundary layer and increase of the cylinder drag by about 6% (Fig. 3a). Displacement of the coordinates of jets toward the rear stagnation point of the flow resulted in a decrease of the cylinder drag. The maximum decrease of the coefficient C_x ($\sim 4\%$) was realized in efflux of the jets with coordinates $\alpha_1 \geq 160^\circ$ and $\alpha_2 \leq 200^\circ$. An increase in the size of the inlet window led to additional (less than 1%) decrease of the drag coefficient at all angular coordinates of the outlet windows. For this configuration of the cylinder, a higher efficiency of two jets with coordinates $\alpha_1 = 160^\circ$ and $\alpha_2 = 200^\circ$ manifested itself.

Oscillations of the drag coefficient increased in efflux of jets with the angular coordinates $\alpha_1 < 140^\circ$ and $\alpha_2 > 220^\circ$ the stronger the larger the medium flow rate (the larger the size of the inlet window L). Within the range of angular coordinates $140^\circ \leq \alpha_1 \leq 180^\circ$ and $220^\circ \geq \alpha_2 \geq 180^\circ$, the level of C_x oscillations virtually did not depend on the parameter L (Fig. 3b).

The amplitude of oscillations of the buoyancy force decreased under the effect of jets at all considered angular coordinates of the outlet windows the stronger the higher the flow rate of the issuing medium (Fig. 3c). The largest

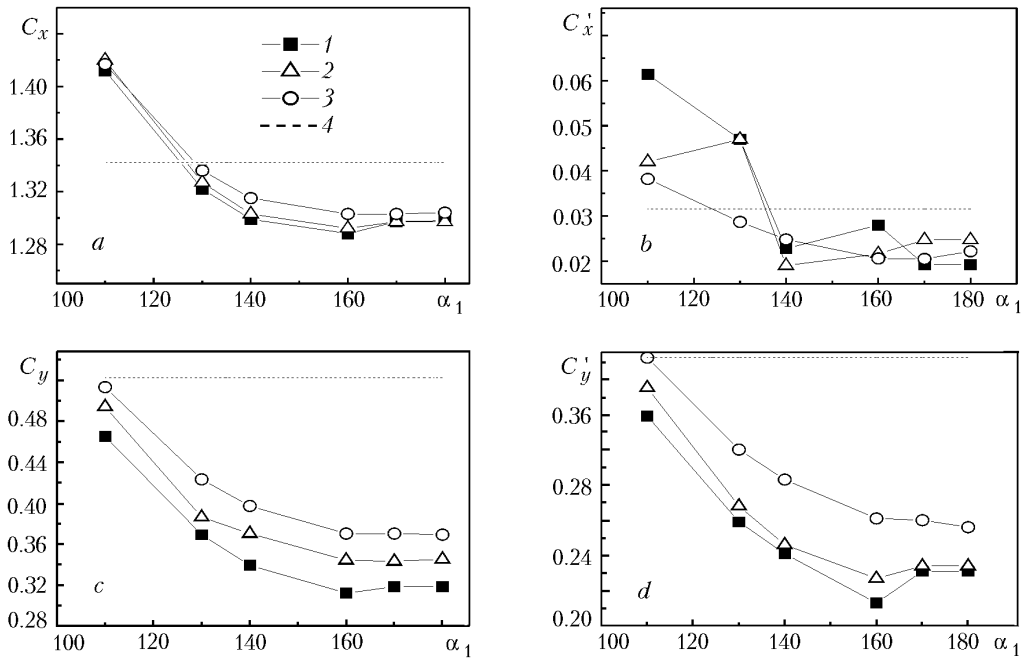


Fig. 3. Variation of the averaged drag coefficient (a), maximum amplitude of the lift coefficient (c) and fluctuations corresponding to them (b, d) depending on the angular coordinate of the issuing jets: 1) $L = 0.3D$, 2) $0.2D$; 3) $0.1D$; 4) basic cylinder. α_1 , deg.

decrease in the amplitude of the coefficient C_y ($\sim 40\%$) took place in blowing-out of the medium from the windows with coordinates $\alpha_1 = 160^\circ$ and $\alpha_2 = 200^\circ$, $\alpha_1 = 170^\circ$ and $\alpha_2 = 190^\circ$, and $\alpha_1 = \alpha_2 = 180^\circ$. The efficiency of the action of two symmetric jets or one central jet is virtually the same.

Root-mean-square oscillations of the amplitude of the lift coefficient decreased with displacement of blowing-out toward the rear stagnation point (Fig. 3d) and reached the smallest values when jets issued through the windows with $\alpha_1 = 160^\circ$ and $\alpha_2 = 200^\circ$, $\alpha_1 = 170^\circ$ and $\alpha_2 = 190^\circ$, and $\alpha_1 = \alpha_2 = 180^\circ$. As the size of the inlet window ($L = 0.2D, 0.3D$) increased, the tendency of the predominance of two jets in attenuation of oscillations of C_y at the angular coordinates $\alpha_1 = 160^\circ$ and $\alpha_2 = 200^\circ$ manifested itself.

For all the cases studied, the Strouhal number attained a value typical of the basic cylinder ($St = 0.184$).

The obtained dependences of mean and oscillatory values of the drag and lift coefficients of the cylinder indicate that for simultaneous decrease of these parameters the jet effect must be localized in the region of $\pm 20^\circ$ relative to the rear stagnation point of the cylinder.

A decrease in l of the outlet windows at a constant value of L of the inlet window is, evidently, accompanied by an increase in the velocity of jet efflux. In this case, the cylinder drag at the smallest angles of the jets ($\alpha_1 = 110^\circ$ and $\alpha_2 = 250^\circ$) decreased, whereas with increase in the angular coordinate of the outlet windows it increased (Fig. 4a–c). Narrow jets suppressed oscillations of the lift coefficient less effectively, and the action of two jets with coordinates $\alpha_1 = 160^\circ$ and $\alpha_2 = 200^\circ$ and $\alpha_1 = 170^\circ$ and $\alpha_2 = 190^\circ$ virtually did not differ from the action of one jet $\alpha_1 = \alpha_2 = 180^\circ$ (Fig. 4d and e).

An increase in the efflux velocity of jets caused a decrease in oscillations of the drag coefficient for the coordinates $\alpha_1 = 110^\circ$ and $\alpha_2 = 250^\circ$ and their increase for the other cases considered, whereas the root-mean-square deviations of the amplitude of the coefficient C_y increased at all coordinates of blowing-out [11].

Variation of the coefficients of pressure and friction on the cylinder surface. The profiles of the coefficients C_p and C_f are obtained by averaging instantaneous distributions in the self-similar region $t > 30$ during the period of variation of values of the coefficient C_y (Fig. 2).

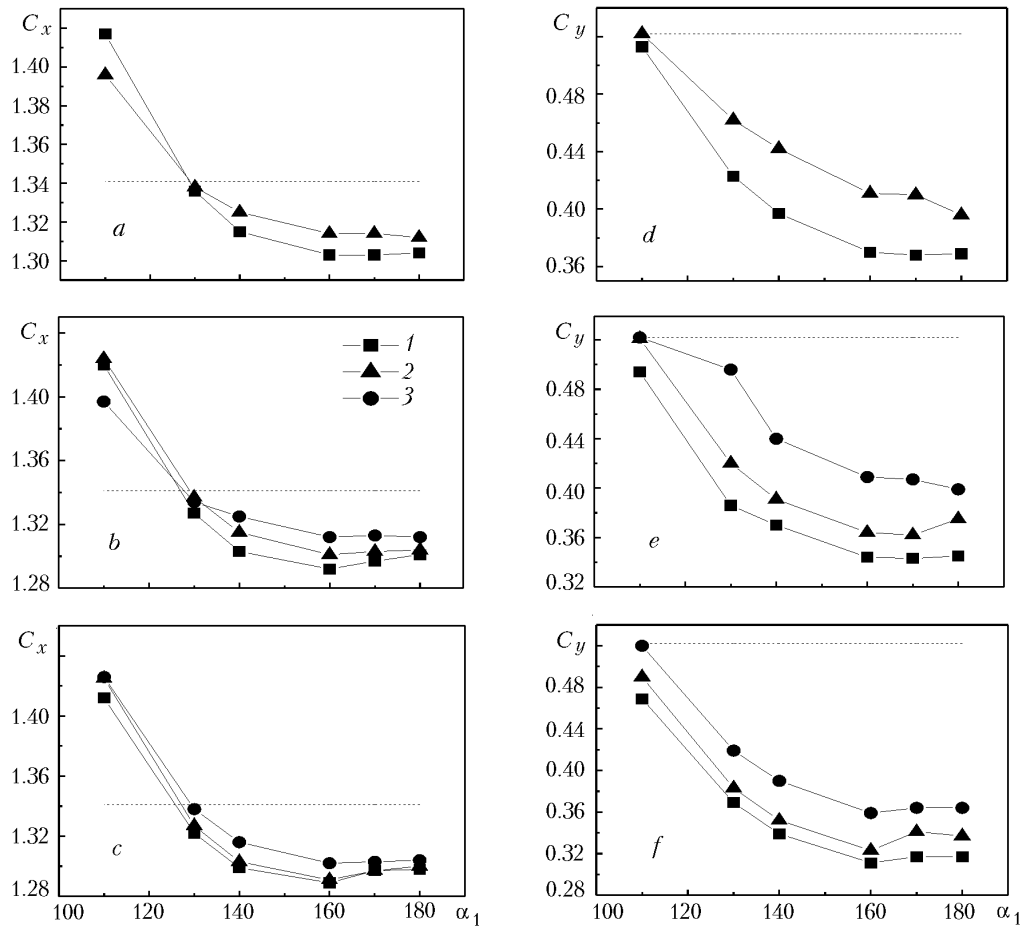


Fig. 4. Variation of the drag and lift coefficients at different velocities of efflux of jets: a, d) $L = 0.1D$, 1) $l_1 = l_2 = 0.05D$; 2) $0.025D$; b, e) $L = 0.2D$, 1) $l_1 = l_2 = 0.1D$; 2) $0.05D$; 3) $0.025D$; c, f) $L = 0.03D$, 1) $l_1 = l_2 = 0.15D$; 2) $0.10D$; 3) $0.05D$; dashed lines indicate the absence of blowing-out. α_1 , deg.

As mentioned earlier, in a laminar flow past the cylinder $Re = 150$, separation of the boundary layer occurs on the coordinate $\sim 150^\circ$, which is in agreement with pressure distributions given in the present study (Fig. 5a–e) [11].

Efflux of jets at all considered positions of the outlet windows caused an increase in pressure on the cylinder surface above the inlet window: $40^\circ \leq \beta \leq \alpha_1$. Pressure decreased behind the outlet windows if they were located in front of the coordinate of boundary-layer separation (Fig. 5a) and increased when the windows were displaced below this coordinate (Fig. 5b–e). The character of variation of pressure distribution over the rear surface of the cylinder allowed us to draw the conclusion that efflux of jets weakly affects the coordinate of boundary-layer separation.

An increase in the velocity of jet efflux (a decrease in the parameter l) revealed a tendency toward some decrease in the pressure increment at all considered sizes of the inlet window. Comparison of pressure distributions in jet efflux from the windows with coordinates $\alpha_1 = 110^\circ$ and $\alpha_2 = 250^\circ$ and $\alpha_1 = 160^\circ$ and $\alpha_2 = 200^\circ$ shows that the difference in the action of jets on the coefficient of pressure manifests itself mainly in the rear region.

Efflux of jets strongly changed the pattern of distributions of oscillations of the coefficient of pressure (Fig. 6a–e). The high level of oscillations was set on the cylinder contour behind the inlet window ($\beta \leq 40^\circ$) and between the coordinates of the outlet windows and the rear stagnation point. Depending on the position of the outlet windows, pressure oscillations within the range of angular coordinates $40^\circ \leq \beta \leq \alpha_1$, α_2 either increased ($\alpha_1 = 110^\circ$ and $\alpha_2 = 250^\circ$) or decreased ($\alpha_1 > 110^\circ$ and $\alpha_2 < 250^\circ$). An increase in the efflux velocity of jets was accompanied by a decrease in the level of generated oscillations.

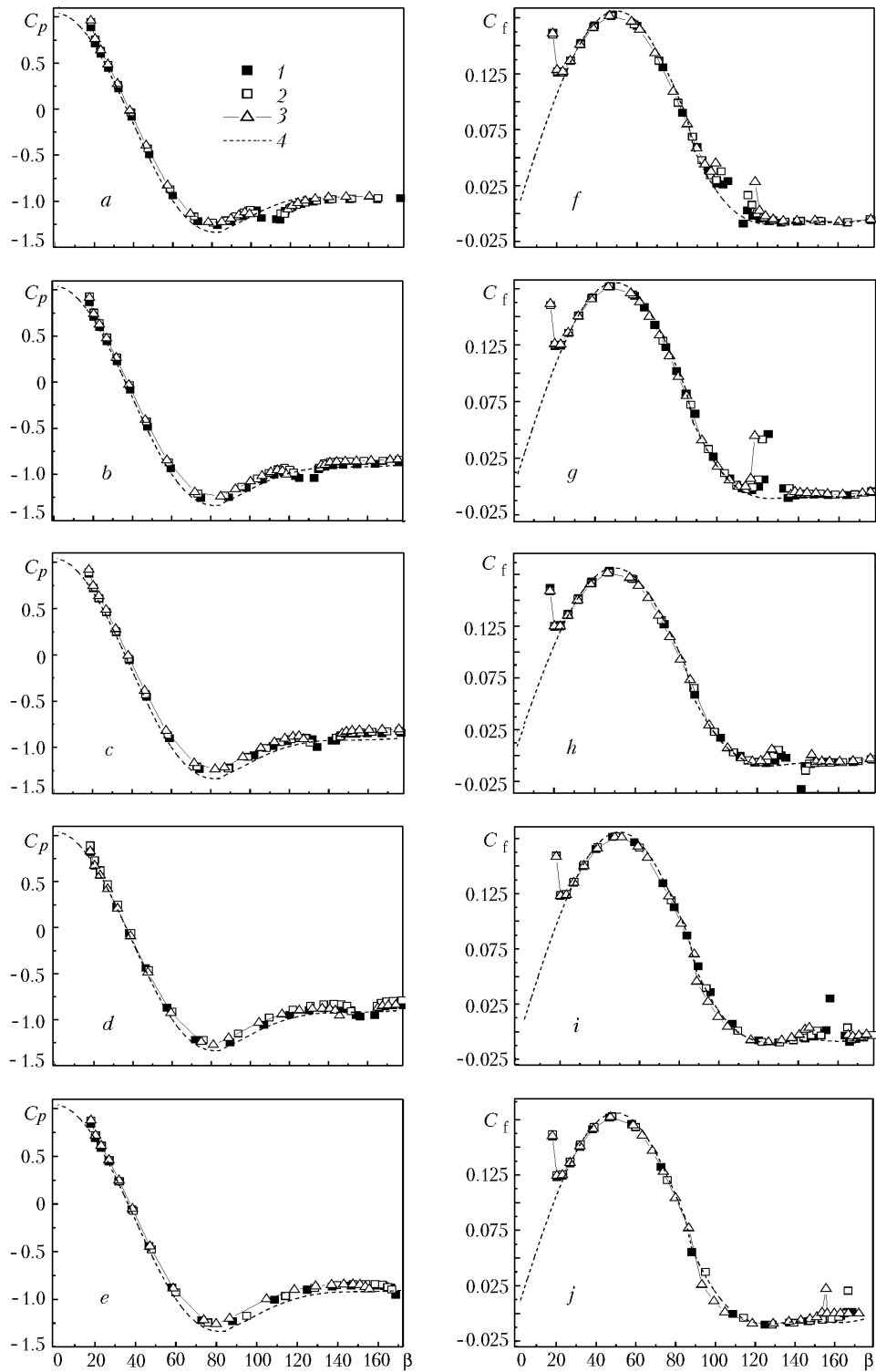


Fig. 5. Variation of the coefficients of pressure and surface friction around the contour of the cylinder with the inlet window $L = 0.3D$: a, f) $\alpha_1 = 110^\circ$, $\alpha_2 = 250^\circ$; b, g) $\alpha_1 = 130^\circ$, $\alpha_2 = 230^\circ$; c, h) $\alpha_1 = 140^\circ$, $\alpha_2 = 220^\circ$; d, i) $\alpha_1 = 160^\circ$, $\alpha_2 = 200^\circ$; 1) $l_1 = l_2 = 0.05D$; 2) $0.10D$; 3) $0.15D$; 4) without blowing-out; e, j) $\alpha_1 = \alpha_2 = 180^\circ$; 1) $l_1 = l_2 = 0.10D$; 2) $0.20D$; 3) $0.30D$; 4) without blowing-out. β , deg.

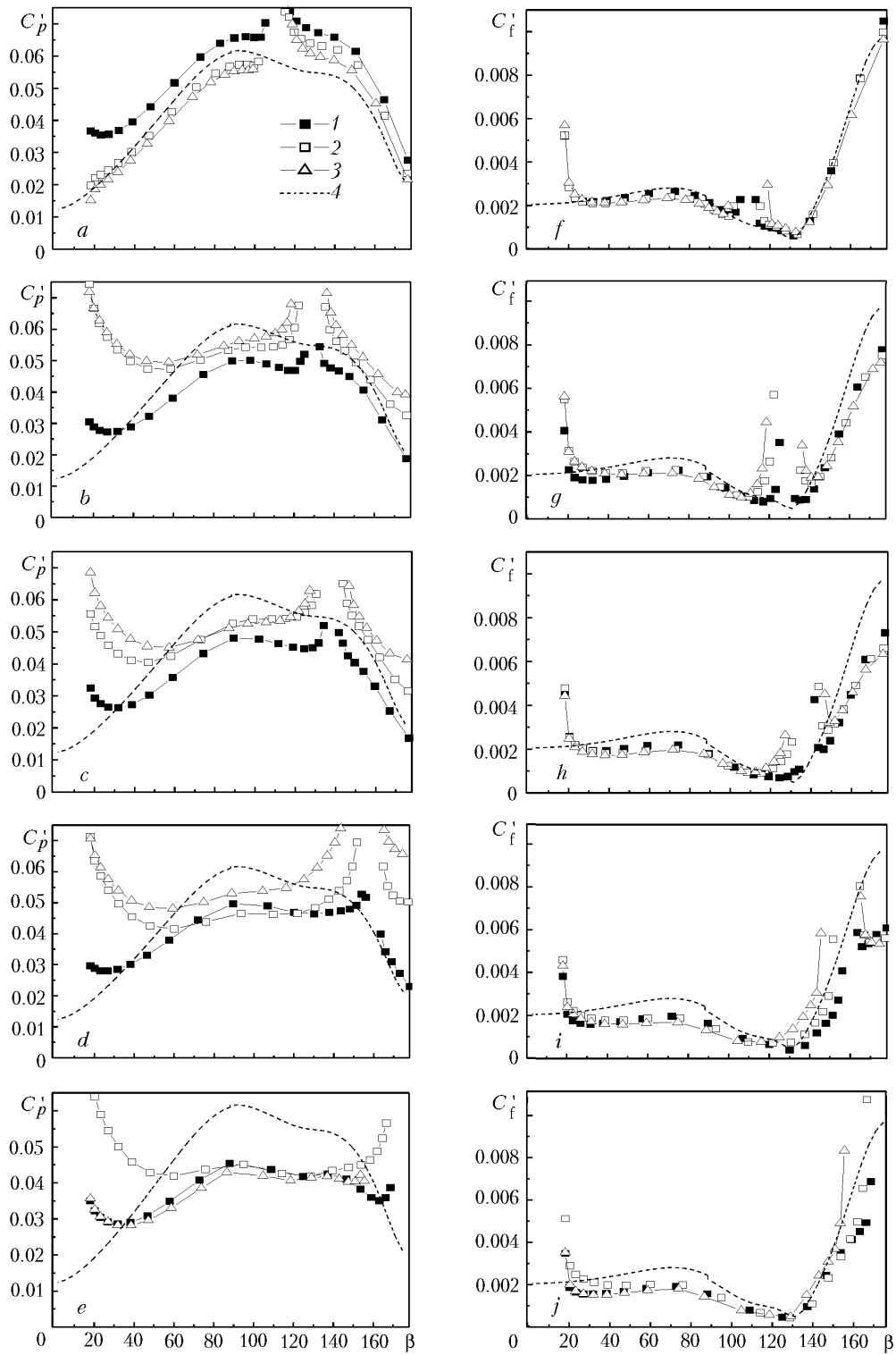


Fig. 6. Variation of oscillations of the coefficients of pressure and surface friction around the contour of the cylinder with the inlet window $L = 0.3D$: a, f) $\alpha_1 = 110^\circ$, $\alpha_2 = 250^\circ$; b, g) $\alpha_1 = 130^\circ$, $\alpha_2 = 230^\circ$; c, h) $\alpha_1 = 140^\circ$, $\alpha_2 = 220^\circ$; d, i) $\alpha_1 = 160^\circ$, $\alpha_2 = 200^\circ$; 1) $l_1 = l_2 = 0.05D$; 2) $0.10D$; 3) $0.15D$; 4) without blowing-out; e, j) $\alpha_1 = \alpha_2 = 180^\circ$; 1) $l_1 = l_2 = 0.10D$; 2) $0.20D$; 3) $0.30D$; 4) without blowing-out. β , deg.

The size of the inlet window exerted the strongest effect on the value of pressure oscillations in the cylinder contour [11]. Thus, for a cylinder with $L = 0.1D, 0.2D$ in the range $\beta \geq 20^\circ$, oscillations exceeded twice the basic ones, whereas at the window size $L = 0.3D$ they increased almost fivefold. The decrease in oscillations in the range of angles $\beta = 80\text{--}140^\circ$ for the configurations $L = 0.1D, 0.2D$ were of an order of 30%, but only 17% for a cylinder with $L = 0.3D$ in efflux of two jets. Distributions of pressure oscillations behind the outlet windows slightly differed from the distributions typical of the basic cylinder if $L = 0.1D, 0.2D$; the values of oscillations increased by 30–35% when $L = 0.3D$. Thus, the cylinder with size of the inlet window $L = 0.3D$ is characterized by the smallest drag and the amplitude of the coefficient C_y and, simultaneously, by high pressure oscillations on most of the contour, which is of interest for heat-transfer problems. Efflux of the central jet for this cylinder caused a decrease in pressure oscillations in the range of angles $\beta = 60\text{--}150^\circ$ by about 34%, as is the case for the configurations with $L = 0.1D, 0.2D$ (Fig. 6e).

Under the action of blowing-out, the averaged coefficient of surface friction decreased in the range of angular coordinates, where an increase in the coefficient of pressure was observed, and increased on the edges of the inlet and outlet windows (Fig. 5f–j). Variation of the velocity of medium blowing-out and the size of the inlet window slightly affected the distributions of this parameter [11].

Oscillations of the coefficient of friction are more sensitive to variations of the conditions of blowing-out (Fig. 6f–j). The calculations showed that oscillations on the front surface of the cylinder decreased as the coordinates of the issuing jets shifted toward the rear stagnation point of the flow. Maximum oscillations of the coefficient C_f on the edges of the outlet windows are caused probably by the separation character of the flow. A considerable decrease of oscillations took place within the range of angular coordinates $\beta = 160\text{--}180^\circ$. An increase in the size of the inlet window was accompanied by a decrease in the level of oscillations in the mentioned range of angles and an increase in oscillations on the edges of this window [11]. The effect of the velocity of jets on this parameter is slight. We note that in efflux of the central jet, oscillations of the coefficient C_f are minimum.

CONCLUSIONS

By numerically investigating the nonstationary influence of blowing-out on a laminar flow past a circular cylinder, we determined the range of optimum angular coordinates of jets at which the maximum decrease of the drag coefficient ($\sim 4\%$) and the amplitude of buoyancy force ($\sim 40\%$) was observed. This range is bounded by the angles $\pm 20^\circ$ relative to the rear stagnation point of flow. The efficiency of the influence of one central or two symmetric jets at the same total flow rate of the medium is virtually the same. An increase in the velocity of the escape of heat, with the flow rate remaining constant, exerted a negative effect on the decrease of the cylinder drag and buoyancy force.

At optimum angles of blowing-out, the coefficient of pressure on the cylinder contour within the range $\beta = 40\text{--}180^\circ$ increased, whereas the coefficient of friction decreased. However, variations of both coefficients are small relative to their distributions for the basic cylinder.

Pressure oscillations on the cylinder contour within the range $\beta = 20\text{--}60^\circ$ increased by a minimum twofold and by $\sim 40\%$ in the section from the outlet windows to the rear stagnation point in blowing-out of two jets. Simultaneously, in the range $\beta = 60\text{--}120^\circ$, oscillations decreased by $\sim 17\%$. Under the effect of efflux of the central jet a lower level of pressure oscillations was generated.

The Strouhal number, as well as the position of the coordinate of boundary-layer separation, remained constant at all studied parameters of inlet and outlet windows. Thus, the observed variations of the parameters of the flow past the cylinder are caused, probably, by transformation of the structure of the near wake, further investigation of which is of interest.

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NOTATION

C_x , coefficient of total drag; C_y , lift coefficient (normal force); C_p , coefficient of pressure; C_f , coefficient of surface friction; D , cylinder diameter; h , height of the internal channel; L , characteristic size, size of the inlet window;

l , size of the outlet window; p , pressure; $Re = DU/\nu$, Reynolds number; t , time; \mathbf{V} , velocity vector; U , longitudinal component of the velocity of oncoming flow; α , angular coordinate of the outlet windows; β , current angular coordinate around the cylinder contour; ν , kinematic viscosity of the medium; ρ , density. Indices: f, friction; t, transposing; x , coordinate along the abscissa axis; y , coordinate along the ordinate axis; cr, critical parameter; $'$, root-mean-square deviation of quantities.

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